**Proceedings of the**
**International Conference on Mechanical Engineering and Renewable Energy 2015**
**(ICMERE2015) 26 – 29 November, 2015, Chittagong, Bangladesh**

**ICMERE2015-PI-116**

# EFFECT OF MAGNETIC FIELD ON NATURAL CONVECTION IN SQUARE CAVITY FILLED WITH $AL_2O_3$–WATER NANOFLUID WITH SQUARE ANNULUS

**Rabiul Hasan Kabir[1,*], Moinuddin Shuvo[2], Khan Md. Rabbi[3], Bibekananda Datta[4], Satyajit Mojumder[5] and Sourav Saha[6]**

[1-6]Department of Mechanical Engineering, Bangladesh University of Engineering & Technology
[1,*]rabiulhasankabir@gmail.com , [2]moinuddinshuvo@gmail.com , [3]khanrabbi92@gmail.com ,
[4]datta.bibek11@gmail.com , [5]sjit018@gmail.com, [6]souravsahame17@gmail.com

***Abstract****- In this paper, natural convection in a square cavity with square cylinder annulus filled with $Al_2O_3$-water nanofluid has been analyzed under external magnetic field. Numerical simulation has been carried out for wide range of Rayleigh number (Ra = $10^4$ ~ $10^6$), Hartmann number (Ha = 0 ~100) and solid volume fraction of nanofluid (φ = 0 ~ 0.15). Galerkin weighted residual method of finite element analysis has been used in this study and grid independency test has been performed to ensure the numerical accuracy of the simulation. Results are shown on the basis of streamlines, isothermal lines. Average temperature of the fluid and Nusselt number (Nu) are plotted to discuss the heat transfer rate. Result of this study indicates that increment of Rayleigh and solid volume fraction of the nanofluid increases the heat transfer rate in a significant way whereas increment of Hartmann number decreases the overall heat transfer rate. Thus better heat transfer rate has been obtained for Ra= $10^6$ at a lower Hartmann number.*

**Keywords:** Magnetic field, Rayleigh number, Annulus, Nanofluid, Nusselt number

## 1. INTRODUCTION

In the present days, heat transfer by natural convection plays an important role in many industries and technology- electronics cooling, heat exchanger, cooling systems [1]. Thus magnetohydrodynamics has grown a special interest as magnetic field is introduced in many heat transfer processes. MHD plays an important role in cooling of nuclear reactors, crystal growth in liquids, geophysics, astrophysics and many more [2]

The plain fluids have lower effective thermal conductivity. So nanoparticles are added to the base fluids. In nanofluids, metal, oxides or carbides can be used as nanoparticle and water, ethylene glycol or oil can be used as base fluid. Together they form a suspension, but some characteristics have to be present in the suspension, such as- the suspension has to be stable, the nanoparticles have to be distributed uniformly throughout the whole base fluids. Typically the nanoparticles that are used have the diameter of 100 mm or less, so their size is close to the size of the base fluid molecules [3].

Mejri et al. [4] showed that heat transfer rate was increased and entropy generation was decreased with the increase of solid volume fraction. He also showed that adding nanoparticles increases heat transfer rate. Abu Nada et al. [5] showed that the average Nusselt number increases with the increment of solid volume friction. Ganji et al. [6] investigated that convection and Nusselt number decrease in presence of magnetic field. Sheikholeslami et al. [7] observed that Nusselt number increases with the increment of solid volume fraction and Rayleigh number where it decreases with the decrease of Hartmann number. Oztop et al. [8] investigated that the heat transfer was decreased with the increase of Hartmann number. Many more studies on the effect of external magnetic field on convection in nanofluids can be found at [9-10]. Many researchers investigated convection phenomena using annulus of different geometries. Gorji-Bandpy et al. [11] investigated MHD free convection using eccentric semi-annulus. Sankar et al. [12] studied vertical cylinder annulus for axial and radial magnetic field effect. MHD natural convection using horizontal cylinder annulus can be found at [13-14].

In the present study, the effect of magnetic field on natural convection in a square cavity filled with $Al_2O_3$ with a square annulus inside is the subject of observation. Here, Rayleigh number and Hartmann number are assumed as parameters. Rayleigh number is varied from $Ra = 10^5$ to $Ra = 10^6$ and Hartmann number is varied from $Ha = 20$ to $Ha = 60$.

## 2. PROBLEM SPECIFICATION

The physical model of the specified problem has been shown in the Fig.1. The length of the walls of the square cavity is $L$. The left wall and the right wall have been kept adiabatic. The bottom wall is kept at high temperature by being heated ($T = T_h$) and the top wall is kept at low temperature ($T = T_c$) where $T_c < T_h$. There is a square annulus inside the cavity whose sides have a length of *0.2L* and the cavity surfaces have a distance of *0.4L* from the annulus walls. The annulus walls are assumed to be adiabatic. A magnetic field with strength $B_o$ is applied horizontally from right side to the left side of the cavity. The effect of gravity is shown along the negative Y axis. $Al_2O_3$-water nanofluid has been considered as the working fluid. Heat transfer by radiation and viscous dissipation effect are assumed to be negligible for simplifying the investigation.

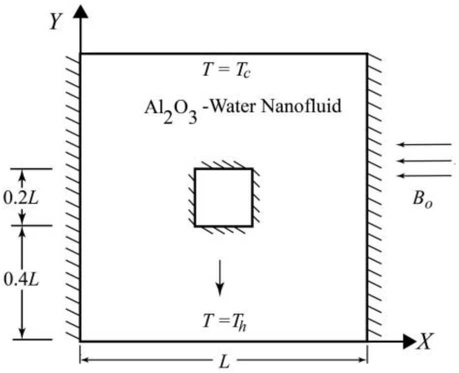


Fig.1: Schematic diagram of a square cavity filled with $Al_2O_3$-water nanofluid with a square annulus.

## 3. MATHEMATICAL FORMULATION

The governing equations of the problem specification are continuity, momentum and energy equations. Boussinesq approximation is assumed to hold true for buoyancy effect. Under these assumptions, the governing equations for the steady, two-dimensional laminar and incompressible MHD convection flow are expressed as follows:

$$\frac{\partial(U\delta)}{\partial X}+\frac{\partial(U\delta)}{\partial Y}=\frac{\partial}{\partial X}\left(\Gamma_\delta\frac{\partial\delta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\Gamma_\delta\frac{\partial\delta}{\partial Y}\right)+S_\delta, \qquad (1)$$

Here, $\delta$ signifies non-dimensional dependent variable whereas $\Gamma_\delta$ and $S_\delta$, denote diffusion and source term respectively. Table 1 showcases the summarized form

Table 1. A summary of the terms of the non-dimensional governing equations (1).

| Equation | $\delta$ | $\Gamma_\delta$ | $S_\delta$ |
|---|---|---|---|
| Continuity | $1$ | 0 | 0 |
| U-momentum | $U$ | $\mu_{nf} / \rho_{nf}\alpha_f$ | $-\partial P/\partial X$ |
| V-momentum | $V$ | $\mu_n f / \rho_{nf}\alpha f$ | $-\partial P/\partial Y + (\rho\beta)_{nf}RaPr\Theta / (\rho_{nf}\beta f) - Ha^2PrV$ |
| Energy | $\Theta$ | $\alpha_{nf} / \alpha_f$ | 0 |

Scales are obtained to find the non-dimensional form of the governing equations which can be defined as-

$$X=\frac{x}{H},Y=\frac{y}{H},U=\frac{u}{u_p},V=\frac{v}{u_p},P=\frac{p}{\rho_{np}u_p^2},\Theta=\frac{T-T_c}{T_h-T_c}. \qquad (2)$$

The non-dimensional governing parameters Rayleigh number ($Ra$), Prandtl number ($Pr$) and Hartmann number ($Ha$) can be expressed as

$$Gr=\frac{g\beta_f(T_h-T_c)L^3}{\nu_f^2},\Pr=\frac{\nu_f}{\alpha_f},Ha=B_0H\sqrt{\frac{\sigma_{nf}}{\rho_{nf}\nu_f}}. \qquad (3)$$

The thermo-physical characteristic parameters are expressed below-

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_p. \qquad (4)$$

$$(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_p. \qquad (5)$$

$$\sigma_{nf}=(1-\varphi)\sigma_f+\varphi\sigma_p. \qquad (6)$$

$$(\beta\rho)_{nf}=(1-\varphi)(\beta\rho)_f+\varphi(\rho\beta)_p. \qquad (7)$$

The dynamic viscosity of nanofluid is obtained by Brinkman model [15]

$$\mu_{nf}=\mu_f(1-\varphi)^{-0.25}. \qquad (8)$$

The thermal conductivity of the nanofluid is expressed by Maxwell-Garnett model [16] and stated as-

$$k_{nf}=k_f\left[\frac{(k_p+2k_f)-2\varphi(k_f-k_p)}{(k_p+2k_f)+\varphi(k_f-k_p)}\right]. \qquad (9)$$

Local Nusselt number can be expressed as,

$$\overline{Nu}=\frac{hL}{k_f}=-\frac{k_{ff}}{k_f}\frac{\partial\Theta}{\partial n}. \qquad (10)$$

$$\frac{\partial\Theta}{\partial n}=\frac{1}{L}\left(\frac{\partial\Theta}{\partial Y}\right) \qquad (11)$$

The characteristics of heat transfer are obtained by average Nusselt number and can be expressed as-

$$Nu=\frac{1}{s}\int_0^s\overline{Nu}ds. \qquad (12)$$

The non-dimensional stream function can be given by,

$$U=\frac{\partial\psi}{\partial Y},V=-\frac{\partial\psi}{\partial X}. \qquad (13)$$

Boundary conditions illustrated in Fig. 1 can be written in dimensionless forms are presented in Table 2

Table 2. Boundary condition in non-dimensional form

| Boundary wall | Flow field | Thermal field |
|---|---|---|
| Bottom wall | U = 0,V = 0 | $\Theta=1$ |
| Top wall | U = 0,V = 0 | $\Theta=0$ |
| Vertical wall and annulus surface walls | U = 0,V = 0 | $\frac{\partial\Theta}{\partial X}=0$ |

## 4. NUMERICAL PROCEDURE

### 4.1 Grid Independency Test

Grid independency test is performed to examine the numerical results of the study. For the observation, *Ha* = 20, *Ra* = $10^5$, $\phi$ = 0.15 is assumed and the velocity profile of the vertical mid-plane is inspected. 456, 1334, 3224, 6286 and 10212 mesh elements have been examined for the test as shown in Fig.2. For the figure, it is observed that mid-plane X-velocity profiles for observation of 6286 and 10212 mesh elements almost overlap each other. So, 6286 elements have been taken as the optimum grid element for the present study.

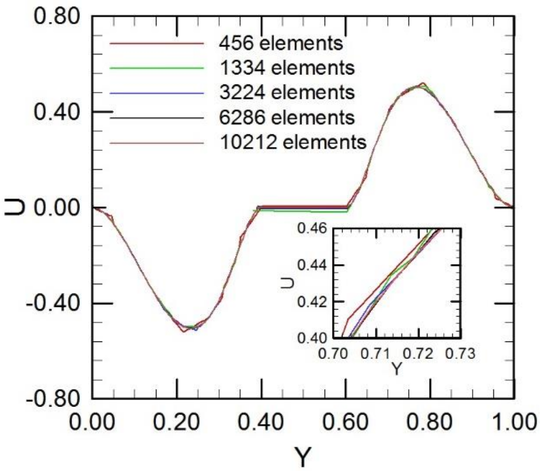


Fig.2: Variation of mid-plane X-velocity with Y-arc length at *Ha* = 20, *Ra* = $10^5$, $\phi$ = 0.15.

### 4.2 Code Validation

The purpose of code validation is to check the reliability of the current code. The present code is compared with the previous study of Ghasemi et al. [17] for validation. For checking the validity average Nusselt number is plotted against solid volume fraction of nanofluid. The following graph shows the comparison of average Nusselt number for solid volume fraction for *Ha* = 0 and *Ha* = 30 at *Ra* = $10^5$ between Ghashemi et al. [17] and the present study. From the graph, it can be said that the values are significantly close to each other.

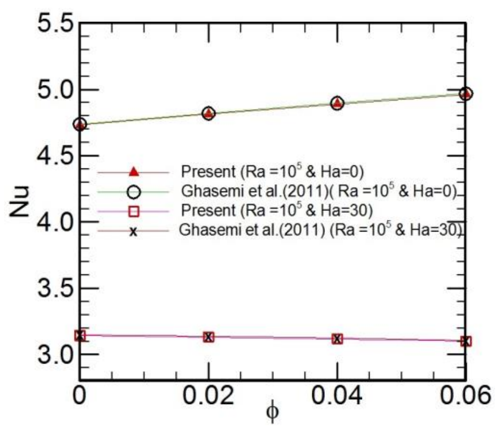


Fig.3: Comparison between the average Nusselt number for different volume fraction Ghasemi et al. [17].

## 5. RESULTS AND DISCUSSION

For observing the effect of magnetic field on natural convection in a square cavity filled with $Al_2O_3$-water nanofluid with a square annulus where the bottom wall of the cavity is being heated, a computational analysis has been done by the finite element method. Rayleigh number and Hartmann number are considered as pertinent parameters. The results are shown by means of streamlines, isothermal lines average Nusselt number and average temperature of the working fluid.

### 5.1 Effect of Hartmann Number on Streamlines and Isotherm Contours:

Fig.4 represents the effect of *Ha* and *Ra* on streamlines. Four vortices are formed because of the presence of square annulus. For Fig.4 (a) where *Ha* = 20 and *Ra* = $10^5$ and for Fig.4 (c) where *Ha* = 60 and *Ra* = $10^5$, it can be observed that the strength of the flow field decreases. This is because of the effect of magnetic field over buoyancy force. For the higher values of *Ha*, the magnetic field also gets higher and it affects and suppresses the buoyancy force. Thus the convection heat transfer decreases. Similar phenomenon is shown in Fig.4 (b) and Fig.4 (d) where *Ha* = 20, *Ra* = $10^6$ and *Ha* = 60, *Ra* = $10^6$ respectively. On the other hand, *Ra* has the opposite effect on streamlines. Fig.4 (a) where *Ha* = 20, *Ra* = $10^5$ and Fig. 4 (b) where *Ha* = 20, *Ra* = $10^6$ show the increase of strength of the flow field with the increase of *Ra*. This happens because with the increment of *Ra*, the buoyancy force gets increased. Thus the convection mode of the heat transfer also increases. This can also be showed from Fig.4(c) with *Ha* = 60, *Ra* = $10^5$ and Fig.4 (d) *Ha* = 60, *Ra* = $10^6$ where strength of the flow field gets increased with the increase of *Ra*.

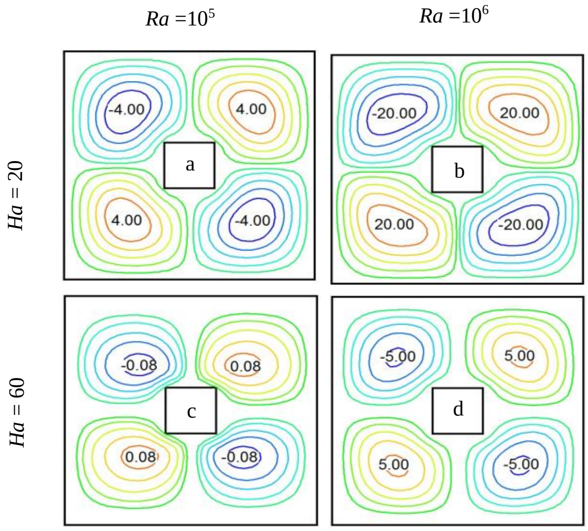


Fig.4: Effect of *Ha* on streamlines when (a), (c) *Ra* = $10^5$ (b), (d) *Ra* = $10^6$ at $\phi$ = 0.15.

The following figures represent the effect of *Ha* and *Ra* on the isotherms. Fig.5 shows that the isothermal lines become less distorted (almost parallel) with the increase of *Ha*. This happens because with the increase of *Ha* as a result of which magnetic force dominates over buoyancy force. Eventually, the conduction mode of heat transfer increases. Similarly this effect of *Ha* on isotherm can be shown from Fig.5 (b) and Fig. 5(d) where *Ha* varies from 20 to 60 and *Ra* = $10^6$. But changes of *Ra* affect differently on the isotherms. Fig.5 (a) with *Ha* = 20, *Ra* = $10^5$ and Fig.5 (b) with *Ha* = 60, *Ra* = $10^6$ represents that the isothermal lines are getting more distorted with the increase of *Ra*. *Ra* increases the buoyancy force which increases the convection mode of heat transfer because buoyancy force becomes more dominating phenomena. Fig.5(c) and Fig. 5(d) show the same effect for the increase of *Ra* on isotherms.

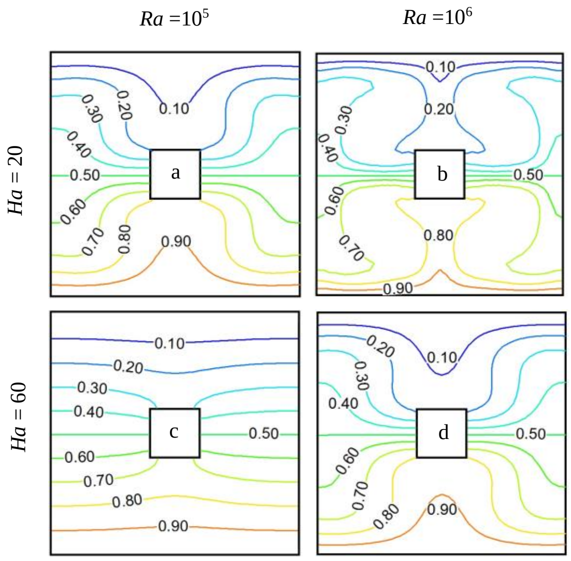


Fig.5: Effect of *Ha* on isotherms when (a), (c) *Ra* = $10^5$ (b), (d) *Ra* = $10^6$ for $\phi$ = 0.15.

**5.2 Effect of Hartmann and Rayleigh Number on Average Nusselt Number and Average Temperature of the Fluid:**

Fig.6 (a) represents the effect of *Ra* on average Nusselt number. Here, average Nusselt number is plotted against *Ra* where $10^4 < Ra < 10^5$.The effect is observed for *Ha* = 20 (red line) and *Ha* = 60 (green line). From the graph, it can be seen that, the average Nusselt number increases with the increase of *Ra*.

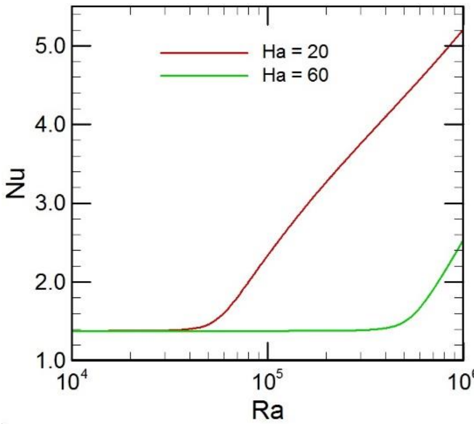


Fig.6 (a): Effect on average Nusselt number with the change of *Ra*.

The increase of average Nusselt number is not that significant when *Ra* varies from $10^4$ to 5 X $10^4$ for *Ha* = 20. At *Ha* = 20, *Ra* > 5 X $10^4$ shows more convection dominated phenomena. For higher magnetic field effect. The dominance of convection mode of transfer over conduction mode can be observed *Ra* > 5 X $10^5$. So the average Nusselt number increases with the increase of *Ra* at a lower *Ha*.

Fig.6 (b) represents the effect of *Ha* on average Nusselt number. Here, the average Nusselt number has been plotted against *Ha* where *Ha* varies from 0 to 100 .The effect is observed for *Ra* = $10^4$ (red line) and *Ra* = $10^5$ (green line). When, *Ra* = $10^4$, no significant change of average Nusselt number can be observed. For *Ra*=$10^5$, the average Nusselt number decreases with the increase of *Ha*. The rate of change is almost constant from *Ha* = 0 to *Ha* = 50. After *Ha* = 50, *Ha* does not emphasize significant impact on the overall heat transfer rate no matter how strong the buoyancy force possess. So, it is shown that the average Nusselt number decreases with the increase of *Ha* at a higher *Ra*.

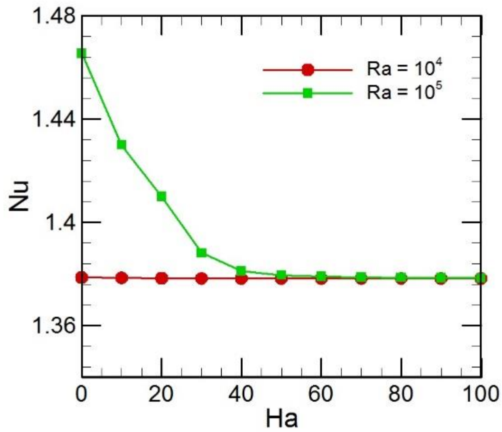


Fig.6 (b): Effect on average Nusselt number with the change of *Ha*

Table 3 shows the effect of *Ra* and *Ha* on the average temperature of the fluid. For inspecting the effect, *Ha* = 20 and *Ha* = 60 have been observed where *Ra* is varied from $10^4$ to $10^6$.

Table 3: Variation of average temperature of the fluid for different *Ha* and *Ra*

| *Ha* | *Ra* | $\Theta_{av}$ |
|---|---|---|
| *Ha* = 20 | 1 x $10^4$ | 0.47999 |
| | 5 x $10^4$ | 0.47998 |
| | 1 x $10^5$ | 0.47996 |
| | 5 x $10^5$ | 0.47993 |
| | 1 x $10^6$ | 0.47990 |
| *Ha* = 60 | 1 x $10^4$ | 1.37825 |
| | 5 x $10^4$ | 1.37847 |
| | 1 x $10^5$ | 1.37899 |
| | 5 x $10^5$ | 1.49673 |
| | 1 x $10^6$ | 2.53705 |

From the Table 3 it can be shown that at lower values of *Ha* (*Ha* = 20) average temperature of the working fluid does not change much. But at higher values of *Ha* (*Ha* = 60) it increases with the increment of *Ra*.

### 5.3 Effect of Solid Volume Fraction of Nanofluid on Average Nusselt Number:

Fig.7 represents the effect of solid volume fraction ($\phi$) on the average Nusselt number (*Nu*). It illustrates that initially, *Nu* decreases with the increase of $\phi$ upto certain value (almost 0.03).Then value of *Nu* increases with the increase of $\phi$ and the rate of increase of *Nu* increases with the increase of $\phi$. The addition of solid nanoparticle ($Al_2O_3$) to the base fluid (water) results in an increase of $\mu_{nf}/\rho_{nf}$ in the diffusion term which describes better buoyancy circulation.

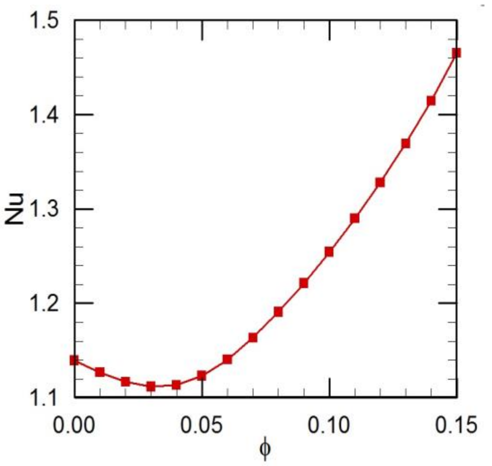


Fig.7: Variation of average Nusselt number *(Nu)* with the solid volume fraction ($\phi$) of the nanofluid.

### 6. CONCLUSION

The effect of magnetic field in a square cavity filled with $Al_2O_3$ –water nanofluid with a square annulus has been studied in the present investigation. From the previous discussion, we can come to the following conclusions-

(1) The strength of the flow field is investigated for the values of *Ha* = 20 and *Ha* = 60. The study shows the strength decreases with the increase of *Ha*. So for better heat transfer by convection, value of *Ha* should be low.

(2) The strength of the flow field is examined for the values of *Ra* = $10^5$ and *Ra* = $10^6$. From observation, it can be concluded that for better convection, higher values of *Ra* is needed as the increase of *Ra* increases the strength of the flow field.

(3) The isothermal lines get less distorted as the value of *Ha* gets higher from 20 to 60. It signifies the increase of conduction mode of heat transfer.

(4) The isothermal lines become more distorted from *Ra* =$10^5$ to *Ra* = $10^6$ .So for the higher values of *Ra*, convection mode of heat transfer gets higher.

(5) The average Nusselt number increases with the increase of *Ra* as observed from *Ra* = $10^4$ to *Ra* = $10^6$.

(6) The average Nusselt number decreases with the increase of *Ha*, where change of *Ha* is observed for 0 to 100.

(7) The average temperature of the fluid decreased of the increase of *Ra* and *Ha* = 20. But it was supposed to increase. This happens because of the presence of external magnetic field. At *Ha* = 60, average temperature increases with the increase of *Ra*.

(8) The average Nusselt number initially decreases with the increase of solid volume fraction (nearly 0.03) and the increases with the increase of the solid volume fraction. The observation is made upto $\phi$ = 0.15.

### 7. ACKNOWLEDGEMENT

The authors would like to take an opportunity to thank Multiscale Mechanical Modeling and Research Network (MMMRN) for their continuous support and thoughtful insight and advice.

### 8. REFERENCES

[1] G. Barakos, E. Mistoulis, "Natural convection flow in a square cavity revisited: laminar and turbulent models with wall functions", *International Journal for Numerical Methods in Fluids,* vol. 18, pp. 695-719, 1994.

[2] A. H. Mahmoudi, I. Pop, M. Shahia, F. Talebia,"MHD natural convection and entropy generation in a trapezoidal enclosure using Cu–water nanofluid", *Computers and Fluids*, vol. 72, pp. 46–62, 2013.

[3] D. D. Ganji, A. Malvandi, "Natural convection of nanofluids inside a vertical enclosure in the presence of a uniform magnetic field", *Powder Technology,* vol. 263, pp. 50–57, 2014.

[4] I. Mejri, A. Mahmoudi, M. A. Abbassi, A. Omri,"Magnetic field effect on entropy generation in a nanofluid-filled enclosure with sinusoidal heating on both side walls", *Powder Technology,* vol. 266, pp. 340–353, 2014.

[5] E. Abu-Nada, A. J. Chamkha,"Effect of nanofluid variable properties on natural convection in enclosures filled with a CuO-EG-Water nanofluid", *International Journal of Thermal Sciences,* vol. 49, pp. 2339-2352, 2010.

[6] M. Sheikholeslami , M. Gorji-Bandpy, D. D. Ganji,S. Soleimani, "Effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cu–water nanofluid using CVFEM", Advanced Powder Technology, vol. 24, no. 6, pp. 980–991, 2013.

[7] M. Sheikholeslami, M. Gorji-Bandpay, D. D. Ganji,"Magnetic field effects on natural convection around a horizontal circular cylinder inside a square enclosure filled with nanofluid", *International Communications in Heat and Mass Transfer,* vol. 39, pp. 978–986, 2012.

[8] H. F. Oztop, M. Oztop, Y. Varol, "Numerical simulation of magnetohydrodynamic buoyancy-induced flow in a non-isothermally heated square enclosure", Communications in Nonlinear Science and Numerical Simulation, vol. 14, pp. 770–778, 2009.

[9] M. Sheikholeslami, M. Gorji-Bandpy, D. D. Ganji,"Numerical investigation of MHD effects on $Al_2O_3$ -water nanofluid flow and heat transfer in a

semi-annulus enclosure using LBM", *Energy*, vol. 60, pp. 501-510, 2013.

[10] H. Nemati, M. Farhadi , K. Sedighi, H. R. Ashorynejad, E. Fattahi, "Magnetic field effects on natural convection flow of nanofluid in a rectangular cavity using the Lattice Boltzmann model", *Scientia Iranica*, vol. 19, no. 2, pp. 303–310, 2012.

[11] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, "MHD free convection in an eccentric semi-annulus filled with nanofluid", *Journal of the Taiwan Institute of Chemical Engineers*, vol. 45, no.4, pp. 1204–1216, 2014.

[12] M. Sankar, M. Venkatachalappa, I.S. Shivakumara, "Effect of magnetic field on natural convection in a vertical cylindrical annulus", *International Journal of Engineering Science*, vol. 44, pp. 1556-1570, 2006,

[13]H. R. Ashorynejad, A.A. Mohamad, M. Sheikholeslami,"Magnetic field effects on natural convection flow of a nanofluid in a horizontal cylindrical annulus using Lattice Boltzmann method", *International Journal of Thermal Sciences*, vol. 64, pp. 240-250, 2013.

[14] M. Sheikholeslami, M. Gorji-Bandpay, D. D. Ganji, "Magnetic field effects on natural convection around a horizontal circular cylinder inside a square enclosure filled with nanofluid", *International Communications in Heat and Mass Transfer*, vol. 39, pp. 978-986, 2012.

[15] H.C. Brinkman, "The viscosity of concentrated suspensions and solution", *The Journal of Chemical Physics*, vol. 20, pp. 571-581, 1952.

[16] J.C. Maxwell, "A Treatise on Electricity and Magnetism", *Oxford University Press, Cambridge, UK*, vol. 2, pp. 54, 1873.

[17] B. Ghasemi, S.M. Aminossadati, A. Raisi, "Magnetic field effect on natural convection in a nanofluid-filled square enclosure", *International Journal of Thermal Sciences*, vol.50, pp. 1748-1756, 2011.

## 9. NOMENCLATURE

| Symbol | Meaning | Unit |
|---|---|---|
| g | acceleration due to gravity | $(ms^{-2})$ |
| k | thermal conductivity | $(W\ m^{-1}K^{-1})$ |
| L | length of the square cavity | (m) |
| Nu | average nusselt number | Dimensionless |
| p | pressure | (Pa) |
| P | dimensionless pressure | Dimensionless |
| Pr | Prandtl number | Dimensionless |
| Ha | Hartmann number | Dimensionless |
| Gr | Grashof number | Dimensionless |
| s | circumference of the domain | |
| T | temperature | (K) |
| u | velocity at x-direction | $(ms^{-1})$ |
| U | dimensionless velocity at x-direction | Dimensionless |
| v | velocity at y-direction | $(ms^{-1})$ |
| V | Dimensionless velocity at x-direction | Dimensionless |
| X | dimensionless distance along x-coordinate | Dimensionless |
| Y | dimensionless distance along y-coordinate | Dimensionless |
| | **Greek symbols** | |
| α | thermal diffusivity | $(m^2s^{-1})$ |
| β | volume expansion coefficient | $(K^{-1})$ |
| φ | solid volume fraction of nanofluid | Dimensionless |
| μ | dynamic viscosity | $(Nsm^{-2})$ |
| ν | kinematic viscosity | $(m^2s^{-1})$ |
| ρ | density | $(kgm^{-3})$ |
| σ | electrical conductivity | $((\Omega\text{-}m)^{-1})$ |
| ψ | stream function | |
| Θ | dimensionless temperature | Dimensionless |
| | **Subscripts** | |
| av | average | |
| h | hot | |
| c | cold | |
| f | base fluid (water) | |
| nf | nanofluid | |
| p | nanoparticle | |